LES simulation of vertical mixing

Emmanuel Hanert

January 22, 2007

1 Explicit LES for stratified flows

Following the work of Özgökmen et al. (2007), we are only going to consider the rather simple eddy viscosity LES models. An example of those is the Smagorinsky model (Smagorinsky, 1963) for calculating the turbulent viscosity coefficient:

$$\nu_T = \nu_{smag} \equiv (c_s \delta)^2 \sqrt{\sum_{i,j} d_{ij}^2}, \tag{1}$$

where $d_{ij} = (\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j})/2$ is an element of the strain rate tensor and δ is the filter length scale, which is typically of the order of the mesh size.

Özgökmen et al. (2007) suggest to modify the Smagorinsky model to take into account the effect of stratification. This is done by multiplying the vertical Smagorinsky viscosity by a dimensionless function of the Richardson number:

$$\nu_{ozg} \equiv f_{ozg}(Ri)\nu_{smag}, \tag{2}$$

where the function f_{ozg} is defined as:

$$f_{ozg}(Ri) = \begin{cases} 1 & \text{for } Ri < 0, \\ \sqrt{1 - \frac{Ri}{Ri_c}} & \text{for } 0 \leq Ri < Ri_c, \\ 0 & \text{for } Ri > Ri_c, \end{cases}$$

where Ri_c is the critical Richardson number, typically $Ri_c = 0.25$. This expression is rather arbitrary but it should certainly go to one when there is no stratification and to zero when stratification is important.

We could also find another way to regulate vertical mixing by considering a simple vertical eddy mixing parameterization like the one introduced by Pacanowski and

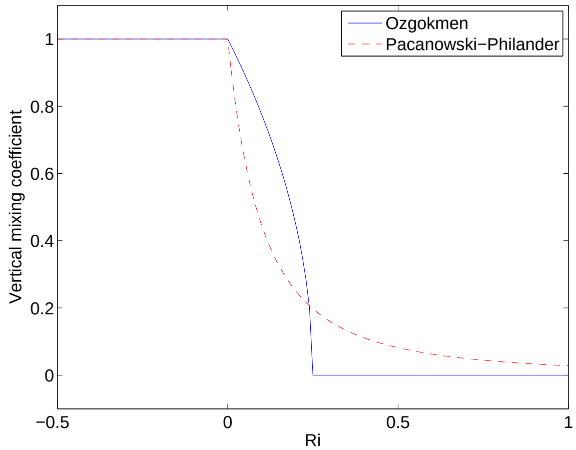

Figure 1: Comparison of the functions $f_{ozg}(Ri)$ and $f_{paca}(Ri)$ used to take the effect of stratification into account.

Philander (1981):

$$\nu_{paca} = \frac{K_u^0}{(1 + \beta_M Ri)^{\alpha_M}} + K_u^*,$$

where the coefficients α_M, β_M, K_u^0 and K_u^* are usually equal to 2, 5, 10^{-2} m^{2}s^{-1} and 10^{-4} m^{2}s^{-1}, respectively. In that expression of the vertical eddy viscosity, K_u^0 corresponds to a sort of upper bound on the vertical eddy viscosity value while K_u^* is a much smaller background value, although larger than the molecular viscosity. The eddy viscosity reduces to K_u^* for large values of Ri. This expression of eddy viscosity suggests the following definition of the weighting function:

$$f_{paca}(Ri) = \begin{cases} 1 & \text{for } Ri < 0, \\ \frac{1}{(1+\beta_M Ri)^{\alpha_M}} & \text{for } Ri > 0, \end{cases}$$

which exhibits the same behaviour as f_{ozg} (see Fig. 1).

We could even go a step further by considering Mellor and Yamada (1982) level 2.5 turbulence closure. In that case, the vertical eddy viscosity reads:

$$\nu_{MY} = lqS_u,$$

where l and q are the turbulence length and velocity scales, respectively. The stability function S_u is a dimensionless function of l, q and the Brunt-Väisälä frequency. This expression of the eddy viscosity suggests the following weighting function:

$$f_{MY}(Ri) = \begin{cases} 1 & \text{for } Ri < 0, \\ \frac{lqS_u}{K_u^0} & \text{for } Ri > 0, \end{cases}$$

where K_u^0 is again an upper bound on the vertical eddy viscosity value and should be of the order of 10^{-2} m^{2}s^{-1}.

2 Some comments about implicit LES

An other approach in LES modelling, called implicit LES modelling, does not rely on filtering the governing equations. Instead, it is based on "smart" non-linear numerical methods that incorporate the subgrid-scale physics into the numerics. Such an approach is usually justified by the fact that the model and the numerics cannot be decoupled unless the flow is fully resolved. This is rarely the case in practice.

The links between implicit and explicit LES can be illustrated by considering the 1D advection equation:

$$\frac{\partial c}{\partial t} + w\frac{\partial c}{\partial z} = 0,$$

where the velocity w is supposed to be positive. A first order finite difference upwind discretization of that equation reads:

$$\frac{c_i^{n+1} - c_i^n}{\Delta t} + w_i\frac{c_i^n - c_{i-1}^n}{\Delta z} = 0.$$

It is well known that the upwind advection term can be rewritten as a centered (non dissipative) advection term plus a diffusion term. The upwind finite difference discretization is thus equivalent to the following one:

$$\frac{c_i^{n+1} - c_i^n}{\Delta t} + w_i\frac{c_{i+1}^n - c_{i-1}^n}{2\Delta z} = \frac{1}{2}\frac{w_i}{\Delta z}\Delta z^2\frac{c_{i+1}^n - 2c_i^n + c_{i-1}^n}{\Delta z^2}.$$

It can be seen that the numerical eddy viscosity is quite similar to the Smagorinsky viscosity, i.e. it is proportional to the square of the element size and to a quantity similar to the shear of the flow. In that respect, this first order upwind scheme contains a (very) simple subgrid-scale model[1]. This very crude upwind model is obviously too dissipative but we could think of higher order upwind schemes that could introduce just the right amount numerical diffusion to filter only the unresolved scales.

The remaining question is to see whether such scheme could take the effect of stratification into account. This could be achieved by having a variable degree of upwinding depending on stratification. Upwinding should increase when stratification decreases in order to have more mixing. Hence, the following vertical advection scheme could be used:

$$\frac{c_i^{n+1} - c_i^n}{\Delta t} + w_i\left[f(Ri)\frac{c_i^n - c_{i-1}^n}{\Delta z} + (1 - f(Ri))\frac{c_{i+1}^n - c_{i-1}^n}{2\Delta z}\right] = 0.$$

[1]This also shows that Smagorinsky's model is a very crude and very dissipative LES model. Using that LES model should be seen as a first attempt but certainly not a definitive choice.

In that case, the amount of upwinding is maximal when there is no stratification ($f = 1$) and decreases with increasing stratification ($f \to 0$). That discretization is equivalent to the following one:

$$\frac{c_i^{n+1} - c_i^n}{\Delta t} + w_i \frac{c_{i+1}^n - c_{i-1}^n}{2\Delta z} = \frac{1}{2} f(Ri) \frac{w_i}{\Delta z} \Delta z^2 \frac{c_{i+1}^n - 2c_i^n + c_{i-1}^n}{\Delta z^2}.$$

This is again quite similar to the kind of scheme suggested by Özgökmen et al. (2007). We could use any of the previously defined vertical mixing functions for f.

References

Mellor, G.L., Yamada, T., 1982. Development of a turbulence closure model for geophysical fluids problems. Review of Geophysics and Space Physics 20, 851–875.

Özgökmen, T.M., Iliescu, T., Fischer, P.F., Srinivasan, A., Duan, J., 2007. Large eddy simulation of stratified mixing in two-dimensional dam-break problem in a rectangular enclosed domain. Ocean Modelling 16, 106–140.

Pacanowski, R.C., Philander, S.G.H., 1981. Parametrization of vertical mixing in numerical models of tropical oceans. Journal of Physical Oceanography 11, 1443–1451.

Smagorinsky, J., 1963. General circulation experiments with the primitive equations. Monthly Weather Review 91, 99–164.